Class XI Session 2023-24
Subject - Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. The value of $\sin \frac{\pi}{10} \sin \frac{13\pi}{10}$ is **[1]**

[Hint: Use sin 18° = $\frac{\sqrt{5}-1}{4}$ and cos 36° = $\frac{\sqrt{5}+1}{4}$]

a) $\frac{1}{2}$ b) $-\frac{1}{2}$

c) $-\frac{1}{4}$ d) 1

2. A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by: x R y ⇔ x is relatively prime to y. Then domain of R is **[1]**

a) {2, 3, 4, 5} b) {3, 5}

c) {2, 3, 5} d) {2, 3, 4}

3. A person write 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is **[1]**

a) $\frac{1}{4}$ b) $\frac{23}{24}$

c) $\frac{15}{24}$ d) $\frac{11}{24}$

4. If y = 1 + $\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$ + ..., then $\frac{dy}{dx}$ = **[1]**

a) y^2 b) y + 1

c) y d) y - 1

5. If p be the length of the perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$, then **[1]**

a) None of these b) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

c) $p^2 = a^2 + b^2$ d) $p^2 = \frac{1}{a^2} + \frac{1}{b^2}$

6. If A = {1, 2, 3, 4, 5, 6} then the number of proper subsets is **[1]**

a) 63
b) 36
c) 64
d) 25

7. If z is any complex number, then $\frac{z-\bar{z}}{2i}$ is **[1]**

a) either 0 or purely imaginary
b) purely imaginary
c) purely real
d) none of these

8. The minimum value of sin x + cos x is **[1]**

a) $-2\sqrt{2}$
b) $\sqrt{2}$
c) 0
d) $-\sqrt{2}$

9. The solution set for: $\frac{|x|-1}{|x|-2} \geq 0, x \neq \pm 2$ **[1]**

a) (-2, 2)
b) $(-\infty, -2) \cup (-1, 1) \cup (2, \infty)$
c) $(-\infty, -2) \cup (2, \infty)$
d) $(-1, 2) \cup (3, \infty)$

10. $\left\{\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right)\right\}$ = ? **[1]**

a) $\frac{1}{\sqrt{2}} \sin A$
b) $2 \sin \frac{\pi}{8}$
c) sin A
d) $\frac{1}{2} \sin A$

11. If A ∩ B = B then **[1]**

a) A = ϕ
b) B = ϕ
c) none of these
d) B ⊆ A

12. The 17th term of the GP 2, $\sqrt{8}$, 4, $\sqrt{32}$... is **[1]**

a) 256
b) $256\sqrt{2}$
c) $128\sqrt{2}$
d) 512

13. $\{C_1 + 2C_2 + 3C_3 + ... + nC_n\}$ = ? **[1]**

a) $(n - 1) \cdot 2^n$
b) $n \cdot 2^n$
c) $(n + 1) \cdot 2^n$
d) $n \cdot 2^{n-1}$

14. The solution set for (x + 3) + 4 > -2x + 5: **[1]**

a) none of these
b) $\left(\frac{-2}{3}, \infty\right)$
c) (- ∞, - 2)
d) (2, ∞)

15. Which set is the subset of all given sets? **[1]**

a) {1}
b) {0}
c) {1, 2, 3, 4}
d) { }

16. The values of cot $\frac{\pi}{3}$, cot $\frac{\pi}{4}$, cot $\frac{\pi}{6}$ are in **[1]**

a) GP
b) None of these
c) HP
d) AP

17. The multiplicative inverse of (-2 + 5i) is **[1]**

a) $\left(\frac{-2}{29}+\frac{5}{29}i\right)$
b) $\left(\frac{2}{29}+\frac{5}{29}i\right)$
c) $\left(\frac{-2}{29}-\frac{5}{29}i\right)$
d) $\left(\frac{2}{29}-\frac{5}{29}i\right)$

18. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, then r is equal to **[1]**

a) 3
b) 5
c) 2
d) 4

19. **Assertion (A):** The expansion of (1 + x)^n = $n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**

Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. Consider the following data **[1]**

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Assertion (A): The variance of the data is 45.8.

Reason (R): The standard deviation of the data is 6.77.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Let A = {1, 2, 3, 4, 5, 6}. Let R be a relation on A defined by R = {(a, b): a, b ∈ A, b is exactly divisible by a}. Find the range of R. **[2]**

OR

Find the domain and the range of the real function: f(x) = $\frac{x^2-16}{x-4}$

22. If $\lim\limits_{x\to a}\frac{x^3-a^3}{x-a} = \lim\limits_{x\to 1}\frac{x^4-1}{x-1}$, find all possible values of a. **[2]**

23. If the distance between the foci of an ellipse is equal to the length of the latus-rectum, write the eccentricity of the ellipse. **[2]**

OR

Find the equation of hyperbola having Foci $(\pm 3\sqrt{5}, 0)$, the latus rectum is of length 8.

24. Let A = {x : x ∈ N, x is a multiple of 3} and B = {x : x ∈ N and x is a multiple of 5}. Write A ∩ B. **[2]**

25. Find the equation of a line that has y-intercept -4 and is parallel to the line joining (2, -5) and (1, 2). **[2]**

Section C

26. The function f is defined by $f(x) = \begin{cases} 1-x, & x<0 \\ 1 & ,x=0 \\ x+1, & x>0 \end{cases}$ **[3]**

Draw the graph of f(x).

27. Solve the inequality $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$ for real x. **[3]**

28. A plane makes intercepts -6,3,4 respectively on the coordinate axes. Find the length of the perpendicular from the origin on it. **[3]**

OR

Verify that (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right-angled triangle.

29. In the expansion of $(x + a)^n$, sums of odd and even terms are P and Q respectively, prove that **[3]**

i. $2(P^2 + Q^2) = (x + a)^{2n} + (x - a)^{2n}$

ii. $P^2 - Q^2 = (x^2 - a^2)^n$

OR

Find the value of $(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4$

30. If $(x + iy)^{1/3} = a + ib$, where x, y, a, b ∈ R, then show that $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$. **[3]**

OR

Find the square roots: 7 - 24i.

31. If U = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11}, A = {2, 4, 7}, B = {3, 5, 7, 9, 11} and C = {7, 8, 9, 10, 11}, then compute **[3]**

i. $(A \cap U) \cap (B \cup C)$

ii. C - B

iii. B - C

iv. (B - C)'

Section D

32. A sample space consists of 9 elementary outcomes $e_1, e_2, ..., e_9$ whose probabilities are **[5]**

$P(e_1) = P(e_2) = .08, P(e_3) = P(e_4) = P(e_5) = 0.1$

$P(e_6) = P(e_7) = .2, P(e_8) = P(e_9) = .07$

Suppose $A = \{e_1, e_5, e_8\}, B = \{e_2, e_5, e_8, e_9\}$

i. Calculate P (A), P (B), and P (A ∩ B)

ii. Using the addition law of probability, calculate P (A ∪ B)

iii. List the composition of the event A ∪ B, and calculate P (A ∪ B) by adding the probabilities of the elementary outcomes.

iv. Calculate P ($\bar{B}$) from P ($\bar{B}$), also calculate P ($\bar{B}$) directly from the elementary outcomes of $\bar{B}$

33. Evaluate the following limits: $\lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7+2x}-(\sqrt{5}+\sqrt{2})}{x^2-10}$. **[5]**

OR

Evaluate : $\lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7-2x}-(\sqrt{5}-\sqrt{2})}{x^2-10}$

34. If S be the sum, P be the product and R be the sum of reciprocals of n terms in a G.P, prove that $P^2 = \left(\frac{S}{R}\right)^n$. **[5]**

35. Prove that: tan 20° tan 30° tan 40° tan 80° = 1 **[5]**

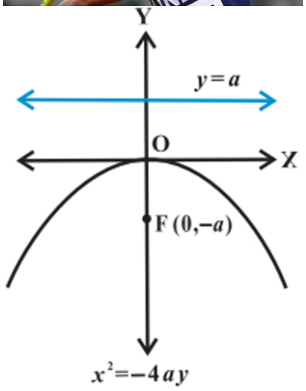
OR

If A + B + C = π, prove that $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = 1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



(i) Name the shape of path followed by a javelin. If equation of such a curve is given by $x^2 = -16y$, then find the coordinates of foci.

(ii) Find the equation of directrix and length of latus rectum of parabola $x^2 = -16y$.

(iii) Find the equation of parabola with Vertex (0,0), passing through (5,2) and symmetric with respect to y-axis and also find equation of directrix.

OR

Find the equation of the parabola with focus (2, 0) and directrix x = -2 and also length of latus rectum.

37. **Read the text carefully and answer the questions:** **[4]**

You are given the following grouped data.

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

(i) Find the difference between mean and median.

(ii) Find the mean deviation about the median.

(iii) Find the mean deviation about the mean.

OR

Mean of the grouped data is

1. 7.0
2. 7.5
3. 8.0
4. 8.5

38. **Read the text carefully and answer the questions:** **[4]**

One evening, four friends decided to play a card game Rummy. Rummy is a card game that is played with decks of cards. To win the rummy game a player must make a valid declaration by picking and discarding cards from the two piles given. One pile is a closed deck, where a player is unable to see the card that he is picking, while the other is an open deck that is formed by the cards discarded by the players. To win at a rummy card game, the players have to group cards invalid sequences and sets.

In rummy, the cards rank low to high starting with Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace, Jack, Queen, and King each have 10 points. The remaining cards have a value equal to their face value. For example, 5 cards will have 5 points, and so on.

Four cards are drawn from a pack of 52 playing cards, then:

(i) In how many ways can four cards are drawn from a pack of 52 playing cards?

(ii) In how many ways can four cards are drawn from a pack of 52 playing cards such that all 4 cards are from same suit?

Solution

Section A

1.

(c) $-\frac{1}{4}$

Explanation: We have sin $\frac{\pi}{10}$sin $\frac{13\pi}{10}$ $=$ sin $\frac{\pi}{10}$sin$\left(\pi+\frac{3\pi}{10}\right)$

= sin $\frac{\pi}{10}$sin$\left(-\frac{3\pi}{10}\right)$ = -sin 18^{o} sin 54^{o}

= -sin18° sin(90° - 36°)

= -sin18° cos36°

= -$\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)$

= $-\frac{(5-1)}{16}$

= $-\frac{1}{4}$

2. **(a)** {2, 3, 4, 5}

Explanation: Relatively prime numbers are those numbers that have only 1 as the common factor.

So, according to this definition we get to know that (2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7) are relatively prime.

So, R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6) ,(5, 7)}.

Therefore, the Domain of R is the values of x or the first element of the ordered pair.

So, Domain = {2, 3, 4, 5}

3.

(b) $\frac{23}{24}$

Explanation: Total number of ways of placing four letters in 4 envelops = 4! = 24

All the letters can be dispatched in the right envelops in only one way. Therefore, the probability that all the letters are placed in the right envelops is $\frac{1}{24}$

Hence, probability that all the letters are not placed in the right envelops = 1 - $\frac{1}{24}=\frac{23}{24}$

4.

(c) y

Explanation: y = 1 + $\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}$ + ...

Differentiating both sides with respect to x, we get $\frac{dy}{dx}=\frac{d}{dx}\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^2}{3!}+\dots\right)$

= $\frac{d}{dx}(1)+\frac{d}{dw}\left(\frac{x}{11}\right)+\frac{d}{dw}\left(\frac{x^2}{2!}\right)+\frac{d}{dw}\left(\frac{x^3}{3!}\right)+\frac{d}{dx}\left(\frac{x^4}{4!}\right)$ + ...

= $\frac{d}{dx}(1)+\frac{1}{1!}\frac{d}{dx}(x)+\frac{1}{2!}\frac{d}{dw}\left(x^2\right)+\frac{1}{3!}\frac{d}{dw}\left(x^3\right)+\frac{1}{4!}\frac{d}{dw}\left(x^4\right)$ + ...

= 0 + $\frac{1}{1!}\times$ 1 + $\frac{1}{2!}\times 2\alpha+\frac{1}{3!}\times 3\alpha^2+\frac{1}{4!}\times 4\alpha^3$ + ... (y = $\alpha^2\Rightarrow\frac{dy}{\partial\alpha}$ = nα^{n-1})

= 1 + $\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}$ + ... $\left[\frac{x}{n!}=\frac{1}{(n-1)!}\right]$

= y

$\therefore\frac{dy}{dx}$ = y

5.

(b) $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Explanation: It is given that p is the length of the perpendicular from the origin on the line

$\frac{x}{a}+\frac{y}{b}=1$

$\frac{1}{a}x+\frac{1}{b}$ y - 1 = 0

Using the formula of the distance we get,

$$\therefore p=\left|\frac{0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|$$

Squaring both sides,

$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

6. **(a)** 63

Explanation: 63

The no. of proper subsets = 2^n - 1

Here n(A) = 6

In case of the proper subset, the set itself is excluded that's why the no. of the subset is 63. But if it is asked no. of improper or just no. of subset then you may write 64

So no. of proper subsets = 63

7.

(c) purely real

Explanation: Let z = x + iy

Then $\bar{z}$ = x - iy

$\therefore$ z - $\bar{z}$ = (x + iy) - (x - iy) = 2iy

Now $\frac{z-\bar{z}}{2i}$ = y

Hence $\frac{z-\bar{z}}{2i}$ is purely real.

8.

(d) $-\sqrt{2}$

Explanation: Let $f(x) = \sin x + \cos x$

$\therefore f'(x) = \cos x - \sin x$

$\Rightarrow f''(x) = -\sin x - \cos x$

Now, $f'(x) = 0$

$\Rightarrow \cos x - \sin x = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1$

$\Rightarrow x = n\pi + \frac{\pi}{4}$, n $\in$ z

At x $= \pi + \frac{\pi}{4}$,

f'' (x) $= -\sin\left(\pi + \frac{\pi}{4}\right) - \cos\left(\pi + \frac{\pi}{4}\right)$

$= \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} > 0$

$\therefore$ x $= \pi + \frac{\pi}{4}$ is point of minimum

Minimum value $= \sin\left(\pi + \frac{\pi}{4}\right) + \cos\left(\pi + \frac{\pi}{4}\right)$

$= -\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)$

$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

9.

(b) $(-\infty, -2) \cup (-1, 1) \cup (2, \infty)$

Explanation: Given $\frac{|x|-1}{|x|-2} \geq 0, x \neq \pm 2$

$\frac{|x|-1}{|x|-2} \geq 0$

$\Rightarrow$ |x| - 1 $\geq$ 0 and |x| - 2 $\geq$ 0 or |x| - 1 $\leq$ 0 and |x| - 2 $\leq$ 0 [$\because \frac{a}{b} \geq 0 \Rightarrow$ (a $\geq$ 0 and b $\geq$ 0) or (a $\leq$ 0 and b $\leq$ 0)]

$\Rightarrow$ |x| $\geq$ 1 and |x| $\geq$ 2 or |x| $\leq$ 1 and |x| $\leq$ 2

$\Rightarrow$ |x| $\geq$ 2 or |x| $\leq$ 1 [$\because$ |x| $\geq$ a $\Rightarrow$ x $\geq$ a or x $\leq$ -a and |x| $\leq$ a $\Rightarrow -a \leq x \leq a$]

$\Rightarrow$ x $\geq$ 2 or x $\leq$ -2 or $-1 \leq x \leq 1$

$\Rightarrow x \in (2, \infty)$ or $x \in (-\infty, -2)$ or $x \epsilon (-1, 1)$

$\Rightarrow x \in (2, \infty) \cup (-\infty, -2) \cup (-1, 1)$

10. **(a)** $\frac{1}{\sqrt{2}} \sin A$

Explanation: Given expression $= \frac{1}{2}\left[2\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - 2\sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right)\right]$

$= \frac{1}{2}\left[\left\{1 - \cos\left(\frac{\pi}{4} + A\right)\right\} - \left\{1 - \cos\left(\frac{\pi}{4} - A\right)\right\}\right]$

$= \frac{1}{2}\left[\cos\left(\frac{\pi}{4} - A\right) - \cos\left(\frac{\pi}{4} + A\right)\right] = \frac{1}{2} \times 2\sin\frac{\pi}{4}\sin A = \frac{1}{\sqrt{2}}\sin A$

11.

(d) B ⊆ A

Explanation: A ∩ B = B which means elements of B are in the both sets A and B.

⇒ All the elements of B are contained in the intersection of A and B which is equal to B.

⇒ B ⊆ A.

12.

(d) 512

Explanation: Given GP is 2, $2\sqrt{2}$, 4, $4\sqrt{2}$, ...

Here, a = 2 and $= \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\therefore T_{17} = ar^{16} = 2 \times (\sqrt{2})^{16} = 2 \times 2^8 = 2^9$ = 512

Therefore, the required 17^{th} term is 512.

13.

(d) $n \cdot 2^{n-1}$

Explanation: $C_1 + 2C_2 + 3C_3 + \ldots + nC_n = n + 2 \cdot \frac{n(n-1)}{2} + 3 \cdot \frac{n(n-1)(n-2)}{3!} + \ldots + n$

$= n \,.\, [1 + (n - 1) \frac{(n-1)(n-2)}{2!} + \ldots + 1]$

$= n.[{}^{(n-1)}C_0 + {}^{(n-1)}C_1 + {}^{(n-1)}C_2 + \ldots + {}^{(n-1)}C_{n-1}]$

$= n\,.(1 + 1)^{n-1} = n.2^{n-1}$

14.

(b) $\left(\frac{-2}{3}, \infty\right)$

Explanation: (x + 3) + 4 > -2x + 5

⇒ x + 7 > -2x + 5

⇒ x + 7 + 2x > -2x + 5 + 2x

⇒ 3x + 7 > 5

⇒ 3x + 7 - 7 > 5 - 7

⇒ 3x > -2

⇒ x > $\frac{-2}{3}$

⇒ $x \in \left(\frac{-2}{3}, \infty\right)$

15.

(d) { }

Explanation: { } denoted as null set and Null set is subset of all sets.

16. **(a)** GP

Explanation: $\cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$, $\cot\frac{\pi}{4} = 1$, $\cot\frac{\pi}{6} = \sqrt{3}$

Now, $\frac{1}{\sqrt{3}}, 1, \sqrt{3}$ are in GP with common ratio $\sqrt{3}$.

17.

(c) $\left(\frac{-2}{29} - \frac{5}{29}i\right)$

Explanation: z = (-2 + 5i) ⇒ $z^{-1} = \frac{1}{z} = \frac{1}{(-2+5i)} \times \frac{(-2-5i)}{(-2-5i)} = \frac{(-2-5i)}{(4-25i^2)} = \frac{(-2-5i)}{(4+25)} = \frac{(-2-5i)}{29}$

⇒ $z^{-1} = \left(\frac{-2}{29} - \frac{5}{29}i\right)$

18. **(a)** 3

Explanation: 3r + r + 3 = 15 [$\because {}^nC_x = {}^nC_y \Rightarrow$ n = x + y or x = y]

⇒ 4r + 3 = 15

⇒ 4r = 12

⇒ r = 3.

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1 + x)^n = n_{c0} + n_{c1}x + n_{c2}x^2 \ldots + n_{c_n}x^n$

Reason:

$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2 + ... + {}^nc_n(1)^{n-n}(-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3} + ... (-1)^n n_{c_n}$

Each term will cancel each other

$\therefore (1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: Presenting the data in tabular form, we get

x_i	f_i	f_ix_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	30	**420**			**1374**

N = 30, $\sum_{i=1}^{7} f_ix_i$ = 420, $\sum_{i=1}^{7} f_i(x_i - \bar{x})^2$ = 1374

Therefore, $\bar{x} = \frac{\sum_{i=1}^{7} f_ix_i}{N} = \frac{1}{30} \times 420$ = 14

$\therefore$ Variance $(\sigma^2) = \frac{1}{N}\sum_{i=1}^{7} f_i(x_i - \bar{x})^2$

$\frac{1}{30} \times 1374$ = 458

Reason: Standard deviation $(\sigma) = \sqrt{45.8}$ = 6.77

Section B

21. Given , According to the question,

R= {(a, b): a, b A, b is exactly divisible by a}

A= {1, 2, 3, 4, 5, 6}

Here,

6 is exactly divisible by 1, 2, 3 and 6

5 is exactly divisible by 1 and 5

4 is exactly divisible by 1, 2 and 4

3 is exactly divisible by 1 and 3

2 is exactly divisible by 1 and 2

1 is exactly divisible by 1

Range of relation R = {1, 2 , 3, 4, 5, 6}

OR

Here we are given that, f(x) = $\frac{x^2-16}{x-4}$

Need to find: where the function is defined.

Let, f(x) = $\frac{x^2-16}{x-4}$ = y ....(i)

To find the domain of the function f(x) we need to equate the denominator of the function to 0

Therefore,

x - 4 = 0 or x = 4

It means that the denominator is zero when x = 4

So, the domain of the function is the set of all the real numbers except 4

The domain of the function, $D_{\{f(x)\}} = (-\infty, 4) \cup (4, \infty)$

Now if we put any value of x from the domain set the output value will be either (-ve) or (+ve), but the value will never be 8

So, the range of the function is the set of all the real numbers except 8

The range of the function, $R_{f(x)} = (-\infty, 8) \cup (8, \infty)$

22. We have given that,

$\lim_{x\to a}\left[\frac{x^3-a^3}{x-a}\right] = \lim_{x\to 1}\left[\frac{x^4-1^4}{x-1}\right]$

$\Rightarrow 3a^{3-1} = 4(1)^{4-1}$

$\Rightarrow 3a^2 = 4$

$\Rightarrow a^2 = \frac{4}{3}$

$\Rightarrow a = \pm\frac{2}{\sqrt{3}}$

23. Given that, the distance between the foci of an ellipse is equal to the length of the latus rectum.

i.e. $\frac{2b^2}{a} = 2ae$

$\Rightarrow e = \frac{b^2}{a^2}$

But, $e = \sqrt{1-\frac{b^2}{a^2}}$

Hence, $e = \sqrt{1-e}$

Squaring both sides, we get:

$e^2 + e - 1 = 0$

$\Rightarrow e = \frac{-1\pm\sqrt{1+4}}{2}$ ($\because$ Ecentricity cannot be negative)

$\Rightarrow e = \frac{\sqrt{5}-1}{2}$

OR

Here foci are $(\pm 3\sqrt{5}, 0)$ which lie on x-axis.

So the equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$ foci $(\pm c, 0)$ is $(\pm 3\sqrt{5}, 0)$

$\Rightarrow c = 3\sqrt{5}$

Length of latus rectum $\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$

We know that $c^2 = a^2 + b^2$

$\therefore (3\sqrt{5})^2 = a^2 + 4a$

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow (a + 9)(a - 5) = 0$

$\Rightarrow a = 5$ ($\because a = -9$ is not possible)

Also a = 5

$\Rightarrow b^2 = 4 \times 5 = 20$

Thus required equation of hyperbola is

$\frac{x^2}{25} - \frac{y^2}{20} = 1$

24. According to the question, we can write,

A = {x : x ∈ N, x is a multiple of 3}

= {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, ...}

B = {x : x ∈ N and x is a multiple of 5}

= {5, 10, 15, 20, 25, 30, 35, 40, 45,...}

Thus, we have

A ∩ B = {15, 30, 45, ...}

= {x : x ∈ N, where x is a multiple of 15}

25. Let m be the slope of the required line.

c = y-intercept = -4

Here, it is given that the required line is parallel to the line joining the points (2, -5) and (1, 2)

$\therefore$ Slope = m = $\frac{y_2-y_1}{x_2-x_1} = \frac{2+5}{1-2}$ = -7

Substituting the values of m and c in y = mx + c, we get, y = -7x - 4

⇒ 7x + y + 4 = 0

Therefore, the equation of the required line is 7x + y = 4 = 0

Section C

26. Here it is given that, f(x) = 1 – x, x < 0, this gives

f(– 4) = 1 – (– 4) = 5;

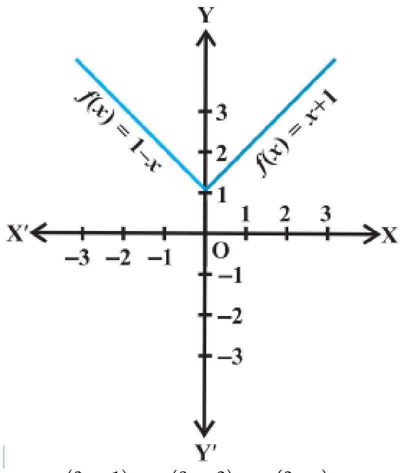
f(– 3) =1 – (– 3) = 4,

f(– 2) = 1 – (– 2) = 3

f(–1) = 1 – (–1) = 2; etc, and f(1) = 2, f (2) = 3, f (3) = 4

f(4) = 5 and so on for f(x) = x + 1, x > 0.

Thus, the graph of the given function f(x) is as shown in figure given below.



27. Here $\frac{(2x-1)}{3} \geqslant \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

$\Rightarrow \frac{2x}{3} - \frac{1}{3} \geqslant \frac{3x}{4} - \frac{2}{4} - \frac{2}{5} + \frac{x}{5}$

$\Rightarrow \frac{2x}{3} - \frac{3x}{4} - \frac{x}{5} \geqslant \frac{-2}{4} - \frac{2}{5} + \frac{1}{3}$

$\Rightarrow \frac{40x-45x-12x}{60} \geqslant \frac{-30-24+20}{60}$

$\Rightarrow \frac{-17x}{60} \geqslant \frac{-34}{60}$

Multiplying both sides by 60, we have

$-17x \geqslant -34$

Dividing both sides by -17, we have

$\frac{-17x}{-17} \leqslant \frac{-34}{-17}$

$\Rightarrow x \leqslant 2$

Thus the solution set is $(-\infty, 2]$

28. According to the question intercepts on the coordinate axes are $(-6, 3, 4)$, then equation of plane will be

$\frac{x}{-6} + \frac{y}{3} + \frac{z}{4} = 1$ or $\frac{x}{-6} + \frac{y}{3} + \frac{z}{4} - 1 = 0$

The distance of a point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$

$$D = \left|\frac{ax_1+by_1+a_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

∴ The distance of origin from given plane

$$\left| = \frac{\left(\frac{-1}{6}\right)\cdot 0+\left(\frac{1}{3}\right)0+\left(\frac{1}{4}\right)0-1}{\sqrt{\left(\frac{-1}{6}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2}}\right|$$

$$= \left|\frac{-1}{\sqrt{\frac{1}{36}+\frac{1}{9}+\frac{1}{16}}}\right| = \left|\frac{-1}{\sqrt{\frac{4+16+9}{144}}}\right| = \left|\frac{-1}{\sqrt{\frac{29}{144}}}\right| = \frac{12}{\sqrt{29}}$$

Required length of the perpendicular from origin to plane is $\frac{12}{\sqrt{29}}$ units.

OR

Let A(0, 7, 10), B(-1, 6, 6) and C(-4, 9, 6) be three vertices of triangle ABC. Then

$$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} = \sqrt{1+1+16} = \sqrt{18}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} = \sqrt{9+9+0} = \sqrt{18}$$

$$AC = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} = \sqrt{16+4+16} = \sqrt{36}$$

$Now, (AB)^2 = 18, (BC)^2 = 18, (AC)^2 = 36$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

Hence, ΔABC is a right-angled triangle.

29. Here $(x + a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + \ldots + {}^nC_na^n$

= P + Q . . . (i)

where $P = {}^nC_0x^n + {}^nC_3x^{n-3}a^3 + \ldots.$

$Q = {}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + \ldots.$

Also $(x - a)^n = {}^nC_0x^n - {}^nCx^{n-1}a + {}^nC_2x^{n-2}a^2 + \ldots + (-1)^{n\,n}C_na^n$ . . . (ii)

= P - Q

(i) Squaring and adding (i) and (ii) we have

$(x + a)^{2n} + (x - a)^{2n} = (P + Q)^2 + (P - Q)^2$

$= P^2 + Q^2 + 2PQ + P^2 + Q^2 - 2PQ$

$= 2P^2 + 2Q^2 = 2(P^2 + Q^2)$

(ii) Multiplying(i) and (ii) we have

$(x + a)^n (x - a)^n = (P + Q)(P - Q)$

$(x^2 - a^2)^n = P^2 - Q^2$

OR

Putting $a^2 = x$ and $\sqrt{a^2 - 1} = y$ we have

$(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4 = (x + y)^4 + (x - y)^4$

$= [{}^4C_0x^4 + {}^4C_1x^3y + {}^4C_2x^2y^2 + {}^4C_3xy^3 + {}^4C_4y^4] + [{}^4C_4x^4 - {}^4C_1x^3y + {}^4C_2x^2y^2 - {}^4C_3xy^3 + {}^4C_4y^4]$

$= 2[{}^4C_0x^4 + {}^4C_2x^2y^2 + {}^4C_4y^4] = 2[x^4 + 6x^2y^2 + y^4]$

$= 2[(a^2)^4 + 6(a^2)^2(\sqrt{a^2 - 1})^2 + (\sqrt{a^2 - 1})^4]$

$= 2[a^8 + 6a^4(a^2 - 1) + (a^2 - 1)^2]$

$= 2[a^8 + 6a^6 - 6a^4 + a^4 - 2a^2 + 1]$

$= 2[a^8 + 6a^6 - 5a^4 - 2a^2 + 1]$

30. We have, $(x + iy)^{1/3} = a + ib$

$\Rightarrow x + iy = (a + ib)^3$ [cubing on both sides]

$\Rightarrow x + iy = a^3 + i^3 b^3 + 3\,iab\,(a + ib)$

$\Rightarrow x + iy = a^3 - ib^3 + i\,3a^2 b - 3ab^2$

$\Rightarrow x + iy = a^3 - 3ab^2 + i\,(3a^2 b - b^3)$

On equating real and imaginary parts from both sides, we get

$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

$\Rightarrow \frac{x}{a} = a^2 - 3b^2$ and $\frac{y}{b} = 3a^2 - b^2$

Now, $\frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$

$= -2a^2 - 2b^2 = -2\,(a^2 + b^2)$

Hence proved.

OR

Let $\sqrt{7 - 24i} = x + iy$. Then

$\sqrt{7 - 24i} = x + iy$

$\Rightarrow 7 - 24i = (x + iy)^2$

$\Rightarrow 7 - 24i = (x^2 - y^2) + 2i\,xy$

$\Rightarrow x^2 - y^2 = 7$ ... (i)

and $2xy = -24$ ... (ii)

Now, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4\,x^2\,y^2$

$\Rightarrow (x^2 + y^2)^2 = 49 + 576 = 625$ [$\because x^2 + y^2 > 0$]

$\Rightarrow x^2 + y^2 = 25$... (iii)

add (i) and (iii), we get

$2x^2 = 32$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

put value of x in (I), we get

$y^2 = 9 \Rightarrow y = \pm 3$

From (ii) we observe that 2xy is negative. So, x and y are of opposite signs.

Hence, $\sqrt{7-24i} = \pm$ (4 - 3i)

31. According to the question, we are given that,

U = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11},

A = {2, 4, 7},

B = {3, 5, 7, 9, 11} and C = {7, 8, 9, 10, 11}

i. (A ∩ U) = {2, 4, 7} ∩ {2, 3, 4, 5, 6, 7, 8, 9, 10, 11} = {2,4, 7}

B ∪ C = {3, 5, 7, 9, 11} ∪ {7, 8, 9, 10, 11} = {3, 5, 7, 8, 9, 10, 11}

$\therefore (A \cap U) \cap (B \cup C)$ = {2, 4, 7} ∩ {3, 5, 7,8,9,10,11} = {7}

ii. C - B = set of all elements which are in C but not in B={7, 8, 9, 10, 11} - {3, 5, 7, 9, 11} = {8, 10}

iii. B - C = {3, 5, 7, 9, 11} - {7, 8, 9, 10, 11} = {3, 5}

iv. We know that for any set A, A' = U-A, therefore, we have, (B - C)' = U - (B - C) = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11} - {3 ,5} = {2, 4, 6, 7, 8, 9, 10, 11}

Section D

32. Given that:Sample Space S = {e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9}

A = {e_1, e_5, e_8} and B = {e_2, e_5, e_8, e_9}

P(e_1) = P(e_2) = .08, P(e_3) = P(e_4) = P(e_5) = .1

P(e_6) = P(e_7) = .2, P(e_8) = P(e_9) = .07

i. To find: P(A), P(B) and P(A ∩ B)

a. A = {e_1, e_5, e_8}

On adding the probabilities of elements of A, we get

P(A) = P(e_1) + P(e_5) + P(e_8)

$\Rightarrow$ P(A) = 0.08 + 0.1 + 0.07 [given]

$\Rightarrow$ P(A) = 0.25

b. B = {e_2, e_5, e_8, e_9}

Similarly, on adding the probabilities of elements of B, we get

P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)

$\Rightarrow$ P(B) = 0.08 + 0.1 + 0.07 + 0.07 [given]

$\Rightarrow$ P(B) = 0.32

c. Now, we have to find P(A ∩ B)

A = {e_1, e_5, e_8} and B = {e_2, e_5, e_8, e_9}

$\therefore$ A ⋂ B = {e_5, e_8}

On adding the probabilities of elements of A ⋂ B, we get

P(A ⋂ B) = P(e_5) + P(e_8)

= 0.1 + 0.07 = 0.17

ii. To find: P(A ∪ B)

a. By General Addition Rule: P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

from part (a), we have

P(A) = 0.25, P(B) = 0.32 and P(A ∩ B) = 0.17

Putting the values, we get

P(A ∪ B) = 0.25 + 0.32 – 0.17 = 0.40

iii. A = {e_1, e_5, e_8} and B = {e_2, e_5, e_8, e_9}

$\therefore$ A ⋃ B = {e_1, e_2, e_5, e_8, e_9}

Again, on adding the probabilities of elements of A ⋃ B, we get

$P(A \bigcup B) = P(e_1) + P(e_2) + P(e_5) + P(e_8) + P(e_9)$

$= 0.08 + 0.08 + 0.1 + 0.07 + 0.07 = 0.40$

iv. To find: $P(\bar{B})$

a. By Complement Rule, we have

$P(\bar{B}) = 1 - P(B)$

$\Rightarrow P(\bar{B}) = 1 - 0.32 = 0.68$

b. Given: $B = \{e_2, e_5, e_8, e_9\}$

$\therefore \bar{B} = \{e_1, e_3, e_4, e_6, e_7\}$

$P(\bar{B}) = P(e_1) + P(e_3) + P(e_4) + P(e_4) + P(e_6) + P(e_{17})$

$= 0.08 + 0.1 + 0.1 + 0.2 + 0.2$ [given] $= 0.68$

33. We have to find the value of $\lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7+2x}-(\sqrt{5}+\sqrt{2})}{x^2-10}$

Re-writing the equation as

$= \lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7+2x}-\sqrt{(\sqrt{5}+\sqrt{2})^2}}{x^2-10}$

$= \lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7+2x}-\sqrt{5+2+2\sqrt{10}}}{x^2-10}$

$= \lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7+2x}-\sqrt{7+2\sqrt{10}}}{x^2-10}$

Now rationalizing the above equation

$= \lim\limits_{x \to \sqrt{10}} \frac{(\sqrt{7+2x}-\sqrt{7+2\sqrt{10}})}{x^2-10} \frac{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

Formula: $(a + b)(a - b) = a^2 - b^2$

$= \lim\limits_{x \to \sqrt{10}} \frac{(7+2x-(7+2\sqrt{10}))}{x^2-10} \frac{(1)}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

$= \lim\limits_{x \to \sqrt{10}} \frac{(2x-2\sqrt{10})}{x^2-10} \frac{(1)}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

$= \lim\limits_{x \to \sqrt{10}} \frac{2(x-\sqrt{10})}{(x+\sqrt{10})(x-\sqrt{10})} \frac{(1)}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

$= \lim\limits_{x \to \sqrt{10}} \frac{2(1)}{(x+\sqrt{10})(1)} \frac{(1)}{(\sqrt{7+2x}+\sqrt{7+2\sqrt{10}})}$

Now we can see that the indeterminant form is removed, so substituting x as $\sqrt{10}$

$= \frac{2}{2\sqrt{10}} \frac{1}{(2\sqrt{7+2\sqrt{10}})}$

$= \frac{1}{2\sqrt{10}} \frac{1}{(\sqrt{7+2\sqrt{10}})}$

OR

We have,

$\lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7-2x}-(\sqrt{5}-\sqrt{2})}{x^2-10}$

$= \lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7-2x}-\sqrt{(\sqrt{5}-\sqrt{2})^2}}{x^2-10} \left(\text{form } \frac{0}{0}\right)$

$= \lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7-2x}-\sqrt{7-2\sqrt{10}}}{x^2-10} \left(\text{form } \frac{0}{0}\right)$

$= \lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7-2x}-\sqrt{7-2\sqrt{10}}}{x^2-10} \times \frac{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}}{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}}$

$= \lim\limits_{x \to \sqrt{10}} \frac{(7-2x)-(7-2\sqrt{10})}{(x-\sqrt{10})(x+\sqrt{10})\left\{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}\right\}}$

$= \lim\limits_{x \to \sqrt{10}} \frac{-2x+2\sqrt{10}}{(x-\sqrt{10})(x+\sqrt{10})\left\{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}\right\}}$

$= \lim\limits_{x \to \sqrt{10}} \frac{-2(x-\sqrt{10})}{(x-\sqrt{10})(x+\sqrt{10})\{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}\}}$

$= \lim\limits_{x \to \sqrt{10}} \frac{-2}{(x+\sqrt{10})\left\{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}\right\}}$

$= \lim_{x \to \sqrt{10}} \frac{-2}{2\sqrt{10}\left\{\sqrt{7-2\sqrt{10}}+\sqrt{7-2\sqrt{10}}\right\}}$

$= \frac{-1}{\sqrt{10}\times 2\times\sqrt{7-2\sqrt{10}}} = \frac{-1}{2\sqrt{10}(\sqrt{5}-\sqrt{2})}$ $[\because (\sqrt{5}-\sqrt{2})^2 = 7-2\sqrt{10}]$

$= \frac{-1}{2\sqrt{10}} \times \frac{(\sqrt{5}+\sqrt{2})}{3} = -\frac{(\sqrt{5}+\sqrt{2})}{6\sqrt{10}}$

34. Let G. P. be a, ar, ar^2, ...

Where r < 1

$S = \frac{a(1-r^n)}{1-r}$

$R = \frac{1}{a} + \frac{1}{ar} + - - + n$

$= \frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^n - 1\right]}{\frac{1}{r}-1}$ $[\because r < 1 \text{ then } \frac{1}{r} > 1$

$= \frac{1}{a} \cdot \frac{1-r^n}{r^n} \cdot \frac{r}{1-r}$

$= \frac{1-r^n}{ar^{n-1}(1-r)}$

P = a.ar.ar^2 ... ar^{n-1}

= an.$r^{1+2+----+(n-1)}$

$= a^n . r^{\frac{n(n-1)}{2}}$

$= a^n . r^{\frac{n(n-1)}{2}}$

L.H.S. = P^2R^n

$= a^{2n} . r^{n(n-1)} \cdot \frac{(1-r^n)^n}{ar^{n-1}(1-r)^n}$

= S^n

$P^2 = \left(\frac{S}{R}\right)^n$

Hence proved.

35. LHS = tan 20° tan 30° tan 40° tan 80°

$= \frac{1}{\sqrt{3}}(\tan 20^\circ \tan 40^\circ \tan 80^\circ)$ $\left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$

$= \frac{(\sin 20^\circ \sin 40^\circ \sin 80^\circ)}{(\cos 20^\circ \cos 40^\circ \cos 80^\circ)\sqrt{3}}$

$= \frac{(2\sin 20^\circ \sin 40^\circ)\sin 80^\circ}{\sqrt{3}(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ}$

Applying

⇒ 2 sin A sin B = cos (A - B) - cos (A + B) and 2 cos A cos B = cos(A + B) + cos (A - B), we get

$= \frac{[\cos(40^\circ-20^\circ)-\cos(20^\circ+40^\circ)]\sin 80^\circ}{[\cos(20^\circ+40^\circ)+\cos(40^\circ-20^\circ)]\cos 80^\circ\sqrt{3}}$

$= \frac{(\cos 20^\circ - \cos 60^\circ)\sin 80^\circ}{\sqrt{3}(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ}$

$= \frac{\left(\cos 20^\circ - \frac{1}{2}\right)\sin 80^\circ}{\sqrt{3}\left(\frac{1}{2}+\cos 20^\circ\right)\cos 80^\circ}$

$= \frac{2\cos 20^\circ \sin 80^\circ - \sin 80^\circ}{\sqrt{3}(\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ)}$

Now,

⇒ 2 sin A cos B = sin (A + B) + sin (A - B)

$= \frac{\sin(80^\circ+20^\circ)+\sin(80^\circ-20^\circ)-\sin 80^\circ}{\sqrt{3}[\cos 80^\circ+\cos(20^\circ+80^\circ)+\cos(80^\circ-20^\circ)]}$

$= \frac{\sin 100^\circ + \sin 60^\circ - \sin 80^\circ}{\sqrt{3}(\cos 80^\circ + \cos 100^\circ + \cos 60^\circ)}$

$= \frac{\sin 100^\circ + \sin 60^\circ - \sin(180^\circ - 100^\circ)}{\sqrt{3}(\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ)}$

$= \frac{\sin 100^\circ + \frac{\sqrt{3}}{2} - \sin 100^\circ}{\sqrt{3}(\cos 80^\circ - \cos 80^\circ + \cos 60^\circ)}$

$= \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}\left(\frac{1}{2}\right)}$ = 1 = RHS

OR

Here it is given that, A + B + C = π and we need to prove that

$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Taking L.H.S, we have

L.H.S = $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}$

$= \frac{1-\cos A}{2}+\frac{1-\cos B}{2}+\frac{1-\cos C}{2}$

$= \frac{1-\cos A+1-\cos B+1-\cos C}{2}$

$= \frac{3-\cos A-\cos B-\cos C}{2}$

Using, cos A + cos A = $2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

L.H.S = $\frac{3-\cos A-\left\{2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right\}}{2}$

$= \frac{3-\cos A-2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2}$

Using, since A + B + C = π

= B + C = 180 - A

And, cos(π – A) = -cosA

L.H.S = $\frac{3-\cos A-2\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2}$

$= \frac{3-\cos A-2\sin\left(\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2}$

Using , cos2A = 1 – 2sin^{2}A

L.H.S = $\frac{3-1+2\sin^2\frac{A}{2}-2\sin\frac{A}{2}\cos\frac{B-C}{2}}{2}$

$= \frac{2-2\sin\frac{A}{2}\left\{\sin\frac{A}{2}-\cos\left(\frac{B-C}{2}\right)\right\}}{2}$

since A + B + C = π

and Using ,

cosA - cosB = $2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$

L.H.S = $\frac{2-2\sin\frac{A}{2}\left\{2\sin\left(\frac{\frac{B+C}{2}+\frac{B-C}{2}}{2}\right)\sin\left(\frac{B+C}{2}-\frac{B-C}{2}\right)\right.}{2}$

$= \frac{2-2\sin\frac{A}{2}\left\{2\sin\left(\frac{\frac{2B}{2}}{2}\right)\sin\left(\frac{\frac{2C}{2}}{2}\right)\right\}}{2}$

Using , since A + B + C = π

L.H.S= $\frac{2-2\sin\frac{A}{2}\left\{2\sin\left(\frac{B}{2}\right)\sin\left(\frac{c}{2}\right)\right\}}{2}$

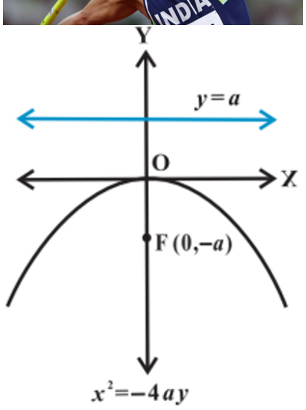
= 1 - $2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{c}{2}$

= R.H.S

Section E

36. **Read the text carefully and answer the questions:**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



(i) The path traced by Javelin is parabola. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

compare $x^2 = -16y$ with $x^2 = -4ay$

$\Rightarrow$ - 4a = -16

$\Rightarrow$ a = 4

coordinates of focus for parabola $x^2 = -4ay$ is (0, -a)

$\Rightarrow$ coordinates of focus for given parabola is (0, -4)

(ii) compare $x^2 = -16y$ with $x^2 = -4ay$

$\Rightarrow$ -4a = -16

$\Rightarrow$ a = 4

Equation of directrix for parabola $x^2 = -4ay$ is y = a

$\Rightarrow$ Equation of directrix for parabola $x^2 = -16y$ is y = 4

Length of latus rectum is 4a = 4 × 4 = 16

(iii) Equation of parabola with axis along y - axis

$x^2 = 4ay$

which passes through (5, 2)

$\Rightarrow$ 25 = 4a × 2

$\Rightarrow 4a = \frac{25}{2}$

hence required equation of parabola is

$x^2 = \frac{25}{2}y$

$\Rightarrow 2x^2 = 25y$

Equation of directrix is y= -a

Hence required equation of directrix is 8y + 25 = 0.

OR

Since the focus (2,0) lies on the x-axis, the x-axis itself is the axis of the parabola.

Hence the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the directrix is x = -2 and the focus is (2,0), the parabola is to be of the form $y^2 = 4ax$ with a = 2.

Hence the required equation is $y^2 = 4(2)x = 8x$

length of latus rectum = 4a = 8

37. **Read the text carefully and answer the questions:**

You are given the following grouped data.

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

(i) Mean - Median

= 7.5 - 7

= 0.5

(ii)

Mean deviation about median = $\frac{\sum_{i=1}^{6} f_i|x_i - \bar{x}|}{\sum_{i=1}^{6} f_i}$

$= \frac{92}{40}$

= 2.3

(iii)

x_i	f_i	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5

	40	300		92

Mean Deviation about the mean

$\bar{x} = \frac{\sum_{i=1}^{6} f_i|x_i - \bar{x}|}{\sum_{i=1}^{6} f_i} = \frac{92}{40}$ = 2.3

OR

Mean = $\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{300}{40}$ = 7.5

38. **Read the text carefully and answer the questions:**

One evening, four friends decided to play a card game Rummy. Rummy is a card game that is played with decks of cards. To win the rummy game a player must make a valid declaration by picking and discarding cards from the two piles given. One pile is a closed deck, where a player is unable to see the card that he is picking, while the other is an open deck that is formed by the cards discarded by the players. To win at a rummy card game, the players have to group cards invalid sequences and sets.

In rummy, the cards rank low to high starting with Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace, Jack, Queen, and King each have 10 points. The remaining cards have a value equal to their face value. For example, 5 cards will have 5 points, and so on.

Four cards are drawn from a pack of 52 playing cards, then:

(i) Here we are selecting 4 cards out of 52 cards

$^nC_r = \frac{n!}{(n-r)!r!}$

$\Rightarrow {}^{52}C_4 = \frac{52!}{(52-4)!4!}$

$\Rightarrow {}^{52}C_4 = \frac{52!}{48!4!} = \frac{52\times51\times50\times49\times48!}{4\times3\times2\times1\times48!}$

$\Rightarrow {}^{52}C_4 = \frac{52\times51\times50\times49}{4\times3\times2\times1}$ = 13 × 17 × 25 × 49

$\Rightarrow {}^{52}C_4$ = 270725

Hence there are 270725 ways to select four cards from pack of 52 cards.

(ii) Since all four cards from same suits so number of possible ways

$^{13}C_4 \times {}^{13}C_4 \times {}^{13}C_4 \times {}^{13}C_4 = 4 \times {}^{13}C_4$

Let

$\Rightarrow {}^{13}C_4 = \frac{13!}{(13-4)!4!}$

$\Rightarrow {}^{13}C_4 = \frac{13!}{9!4!} = \frac{13\times12\times11\times10\times9!}{4\times3\times2\times1\times9!}$

$\Rightarrow {}^{13}C_4 = \frac{13\times12\times11\times10}{4\times3\times2\times1}$ = 13 × 11 × 5

$\Rightarrow {}^{13}C_4$ = 715

Hence required number of combinations

= 4 × 715 = 2860